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PATENT SPECIFICATION

1,004,178

DRAWINGS ATTACHED

Date of Application and filing Complete Specification: Dec. 28, 1962.

No. 48746/62.

Application made in United States of America (No. 163,107) on Dec. 29, 1961.

Complete Specification Published: Sept. 8, 1965.

ndex at acceptance:—**H3** B(5X, 7A, 10X10); **H1** K(1D1, 1D4X, 2S18, 5)

nt. Cl.:—H 03 f // H 01 c

COMPLETE SPECIFICATION

Superconductive Oscillator Circuits

We, INTERNATIONAL BUSINESS MACHINES CORPORATION, a corporation organized and existing under the laws of the State of New York, United States of America, of Armonk, New York 10504, United States of America, (assignees of EUGENE STEWART SCHLIG and HAROLD SOBOL) do hereby declare the invention for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to oscillator circuits and more specifically to such circuits wherein superconductive components are employed.

An oscillator embodying the invention will now be described by way of example with reference to the accompanying drawings, in which:—

Figure 1 illustrates one arrangement of an L—C oscillator according to this invention;

Figures 2*a* and 2*b*, 3*a* and 3*b*, 4, 5, and 6 show curves which help to illustrate the operation of the oscillator circuit of Figure 1;

Figures 7 and 8 illustrate additional L—C oscillator circuits according to this invention;

Figures 9 and 10 illustrate oscillator circuits according to this invention which utilize transmission lines as the frequency determining element, and

Figures 11, 12 and 13 show curves which help to illustrate the operation of the oscillator circuits of Figures 9 and 10.

Reference is made to Figure 1 for a description of a cryogenic oscillator according to this invention wherein the resonant frequency is determined by an L—C circuit. A source of direct current 10 is connected through a variable resistor 12 to a switch 14 as shown. Resistor 12 is sufficiently large compared to the impedance of the rest of the circuit that the current I_s is a substantially constant current. Current from the battery 10 is designated as I_s, and it flows to a junction point 16. Between the junction points 16 and 18 are two parallel circuits. One parallel circuit includes a gate 20 of a cryotron 22, and the other parallel circuit includes a control winding 24 of the cryotron 22 connected in parallel with a current source formed by a resistor and battery 26, and in series with a capacitor 28 and an inductance here represented by coil 30. The current I_s from the battery 10 divides at the junction point 16 into two components I_g and I_t. The current I_g is the current which passes through the gate element 20 of the cryotron 22, and the current I_t is the current which passes through the tank circuit composed of the capacitor 28 and the coil 30.

All interconnecting lines as well as the control winding 24, the condenser 28 and the coil 30 are fabricated of a hard superconductive material, while the gate element 20 is fabricated of a soft superconductive material. The source of current 10 supplies operating current to the circuit shown, and the source current 26 supplies a bias current to the cryotron 22.

When oscillatory currents are desired, the switch 14 is closed and current I_s is supplied to the junction point 16. The quiescent path of the direct current I_s is the parallel branch containing the gate element 20. Assume that a portion of this current I_t is transiently diverted to the junction point 32 in the parallel branch including tank circuit. The current I_t flowing to the junction point 32 and the current I_b flowing to the same point combine in an aiding direction, and they are applied to the control winding 24 of the cryotron 22. These combined currents flow to the junction point 34 at which point the current I_t flows through the condenser 28 and the coil 30 to the junction point 18 while the current I_b flows to the battery 26. The current I_t and the current I_g combine at the junction point 18 and flow as current I_s back to the battery 10. Let it be assumed that the total current through the control winding 24 of the cryotron 22 creates a magnetic field on the gate element 20 which is greater than the critical magnetic field of the gate. This drives the gate element

20 resistive, thereby diverting more of the battery current I_s to the parallel circuit including the tank circuit. This charges the capacitor 28, but as the capacitor 28 acquires a charge, the current I_t is diminished. As soon as the current I_t is reduced to a value such that the sum of the current I_t and the current I_b is less than the critical current of the cryotron 22, the gate element 20 reverts to its superconductive state, and the current I_g again flows through the superconductive gate 20. The resultant wave form of the current in the tank circuit during the period when the gate element 20 is resistive is that of a damped sinusoidal oscillation. The wave form of the current in the tank circuit during the period when the gate element 20 is superconductive is approximately that of an undamped sinusoidal oscillation. The amplitude of the oscillatory energy may build up because the energy supplied from the source of working current 10 during that part of the cycle when the gate element 20 is resistive may be made to be greater than the energy dissipated.

It may be shown that the energy in the resonant circuit is increasing at any instant when the gate element 20 is resistive and the tank circuit current I_t is positive and less than the supply current I_s. This neglects dissipation other than that of the resistive gate and the effects of loading the tank circuit. The bias current is necessary to prevent the gate element 20 from becoming resistive during the negative half cycles. In this connection it is pointed out that when the capacitor 28 is discharging, the tank current I_t opposes the bias current I_b. Thus the bias current I_b is made sufficiently great in amplitude to prevent the oscillatory currents on the negative half cycles from driving the gate element 20 into its resistive state.

Incremental gain, i.e. the value of $\left|\dfrac{dI_g}{dI_c}\right|$ at a specified point on the gain curve, in the cryotron is necessary to switch the gate from its superconductive state to the resistive state, and *vice versa*, for the purpose of maintaining oscillations. The resistance-field transition may be quite abrupt, but absolute abruptness is not essential since other modes of operation may occur where a slow or sloping transition takes place.

Sources of dissipation other than the gate resistance, such as dielectric losses in the capacitor and interconnecting lines and loading of the tank circuit, will somewhat modify the operation of the circuit and the conditions for buildup. Nevertheless, the total waveform of the oscillator circuit is approximately sinusoidal, the deviation from this waveform arising from the existence of resistance in the gate during part of the cycle and its absence during the remainder of the cycle.

At this point it is convenient to analyse the circuit in Figure 1 and demonstrate a proof of its validity as an oscillation generator. For this purpose the direct current I_s is considered energy supplied to the circuit; the instantaneous tank current is designated i_t; the gate current is $I_s—i_t$ which is designated i_g; and the resistance of the gate 20 when switched is arbitrarilly designated R_g. Other symbols are defined as they are introduced subsequently.

In order that oscillation be building up or maintaining itself at any instant, it is essential that the energy supplied be equal to or greater than the energy dissipated by R_g. It is demonstrated hereafter how this affects the circuit parameters.

Energy supplied $\geq$ energy dissipated

$$I_s\, i_g\, R_g \geq i_g^2 R_g$$

If i_g is positive, this relation becomes

$$I_s \geq i_g$$

(If i_g is negative, energy is lost unless I_s is reversed.)

The conditions reduce to

$$\begin{array}{ll} (1) & i_g > 0 \\ \text{or} & I_s > i_t \\ (2) & I_s \geq i_g \\ \text{or} & i_t \geq 0 \end{array}$$

Combined, we see that for buildup or maintenance,

$$0 \leq i_t < I_s$$

is necessary when the gate is resistive.

The above implies that the cryotron be made to switch only when i_t is positive and that the switching point be at a value of i_t less than I_s. The bias current I_b must have a polarity such that it aids i_t in the control winding 24 during the positive swing of i_t. Furthermore, the critical value of i_t, which will be called I_c^1, must be less than I_s. I_c^1 may be defined as

$$I_c^1 = I_c - I_b$$

Here I_c is the actual critical control current of the cryotron at a gate current of $I_s - I_c^1$.

It is helpful to observe the plot of the path of operation of the circuit on a cryotron gain curve, and this is illustrated in Figure 2*a*. The axes represent i_g and i_t as shown, and the major axis of the elliptical gain curve is shifted left by the amount I_b. The locus of operation is the straight line

$$i_g = I_s - i_t$$

which intersects each axis at I_s.

It may be immediately seen that, in order that resistance be introduced for increasing i_t at positive values of i_t and i_g, incremental gain, as defined above, greater than unity is essential.

The letters A through E relate points on the diagram to points on the sinusoidal variation of i_t shown in Figure 2*b*. Note that it is possible with the cryotron shown for i_t to exceed I_c on the positive swing for a lower amplitude than that for which it reaches the critical value, point E, on the negative swing. In this case, amplitude limiting of the oscillation would occur due to the reversal of i_g in the resistive gate. Were point E reached first, limiting would occur due to switching of the gate to a resistive condition when i_t is negative.

The analysis below will assume the former is the case, since limiting by gate current reversal is linear in the sense that it does not involve parameter changes, and so it does not introduce distortion of the waveform. This limiting mechanism is implied in the mathematical treatment below, and it will give rise to the results to a stable value of peak oscillation current.

Since the circuit resistance assumes the constant value zero for a range of values of i_t, and another constant value R_g for the remainder of the range of i_t, a step-by-step linear analysis may be performed. For this purpose reference is made to the waveforms of Figures 3*a* and 3*b*.

First, consider that part of the oscillation cycle shown in Figure 3*a* for which $i_t < I_c^1$ and the gate resistance is zero. The waveform of i_t is constrained to be that of an undamped sine wave since the loop in which it flows contains only L and C. For this part of the cycle i_t is designated I_1 to distinguish the solutions for the two linear steps.

At $t=0$ assume i_1 equals I_o^1 and, since it is desired to obtain a negative $\frac{di_1}{d_t}$ initially, let the initial charge on C be assumed to be a positive value Q_0.

The Laplace transform equation for the circuit is:

$$\frac{1}{CS}\left(Q_o + I_1(S)\right) + LS\,I_1(S) - LI_o^1 = 0$$

$$I_1(S) = I_o^1 \frac{S - \frac{Q_o}{LCI_o^1}}{S^2 + \frac{1}{LC}}$$

By defining $(LC)^{-\frac{1}{2}}$ as ω_0 as is usual, the inverse transform is found to be:

$$\text{(1A)}\quad i_1 = I_o^1\left[\cos\omega_0 t - \frac{Q_o}{I_o^1}\omega_o \sin\omega_0 t\right]$$

or

$$\text{(1B)}\quad i_1 = -I_c^1\left(\left(\frac{\omega_o Q_o}{I_o^1}\right)^2 + 1\right)^{\frac{1}{2}} \sin\left[\omega_0 t - \tan^{-1}\left(\frac{I_o^1}{\omega_o Q_o}\right)\right]$$

The next point of interest on the wave form is that at which i_1 again equals I_o^1 which will occur at time T_1. At time T_1, ( ) $I_o^1 = I_o^1 \cos\omega_o T_1 - Q_o\,\omega_o \sin\omega_o T_1$ which yields

$$\text{(2B)}\quad Q_o = \frac{I_o^1}{\omega_o}\left[\frac{\cos\omega_o T_1 - 1}{\sin\omega_o T_1}\right]$$

The division by $\sin\omega_0 T_1$ eliminates the trivial $T_1 = 0$ solution. This equation relates T_1 to Q_o

Next the charge Q_1 at T_1 is found

$$Q_1 = Q_o + \int_0^{T_1} i_1(t)\,dt$$

$$Q_1 = Q_o + I_o^1 \int_0^{T_1}\left[\cos\omega_0 t - \frac{Q_o}{I_o^1}\omega_o \sin\omega_o t\right] dt$$

$$\text{(3)}\quad Q_1 = Q_o\left[\cos\omega_o T_1 + \frac{I_o^1}{\omega_o Q_o}\sin\omega_0 T_1\right]$$

Substitution (2B) into (3),

$$Q_1 = \frac{I_o^1}{\omega_o}\left(\frac{1-\cos\omega_o T_1}{\sin\omega_o T_1}\right) = -Q_o$$

$$\text{(4)}\quad Q_1 = -Q_o$$

This result might have been obtained directly from the symmetry of the sinusoidal waveform, by observing that the slope at $t=0$ is the negative of that at T_1 and that the charge is proportional to the rate of change of current in a loop containing only L and C.

From the foregoing the initial conditions

have been found for that portion of the cycle shown in Figure 3*b* for which the gate has resistance R_g. For this part of the cycle i_t is designated I_2 to distinguish the solutions for the two linear pieces under consideration. The Laplace transform equation for the current on a new time scale starting from t=0 in Figure 3*b* is:

$$\frac{1}{CS}\left(Q_1 + I(S)\right) + LSI(S) - LI_o^1 + R_g I(S) - \frac{R_g I_o}{S} = 0$$

$$I(S) = I_o^1 \frac{S + \left(\frac{R_g}{L}\frac{I_s}{I_o^1} - \frac{Q_1}{LCI_o^1}\right)}{S^2 + \frac{R_g}{L}S + \frac{1}{LC}}$$

Where

$$a_0 = \frac{R_g}{L}\frac{I_s}{I_o^1} - \frac{Q_1}{LCI_o^1}$$

$$\alpha = \frac{R_g}{2L}$$

$$\beta = \sqrt{\frac{1}{LC} - \frac{R_g^2}{4L^2}}$$

An oscillatory solution is assumed, (that is, $\frac{R^2}{4L^2} < \frac{1}{LC}$) so that the wave form will remain as nearly sinusoidal as possible.

The transform is:

$$I(S) = I_o^1 \frac{S + a_0}{(S+\alpha)^2 + \beta^2}$$

In these terms, the inverse transform for the current i_2 is:

$$\text{(5A)} \quad i_2 = I_o^1 e^{-\alpha t}\left[\cos\beta t + \frac{a_0 - \alpha}{\beta}\sin\beta t\right]$$

or

$$\text{(5B)} \quad i_2 = I_o^1 \frac{\left[(a_0-\alpha)^2 + \beta^2\right]^{\frac{1}{2}}}{\beta} e^{-\alpha t} \sin\left[\beta t + \tan^{-1}\frac{\beta}{a_0 - \alpha}\right]$$

As before, let $i_2 = I_o^1$ (using 5A) at time T_2

$$\text{(6A)} \quad I_o^1 = I_o^1 e^{-\alpha T_2}\left[\cos\beta T_2 + \frac{a_0 - \alpha}{\beta}\sin\beta T_2\right]$$

and solve for the value of Q_1 for which $i_2 = I_o^1$ at T_2

$$\text{(6B)} \quad Q_1 = \frac{\beta I_o^1}{\alpha^2+\beta^2}\frac{\cos\beta T_2}{\sin\beta T_2} - \frac{\beta I_o^1}{\alpha^2+\beta^2}\frac{1}{e^{-\alpha T_2}\sin\beta T_2} - \frac{\alpha I_o^1}{\alpha^2+\beta^2} + \frac{\alpha(2L_s)}{\alpha^2+\beta^2}$$

The charge at time T_2 in Figure 3*b* is:

$$Q_2 = Q_1 + \int_0^{T_2} i_2\, dt = Q_1 + \Delta Q$$

where, using equation (5A),

$$(7)\quad \begin{cases} \Delta Q = \int_0^{T_2} i_2\, dt = Q_1 \left[\frac{\alpha}{\beta} e^{-\alpha T_2} \sin \beta T_2 + e^{-\alpha T_2} \cos \beta T_2 - 1 \right] \\ -2I_s \left[\frac{\alpha_2}{\beta(\alpha^2+\beta^2)} e^{-\alpha T_2} \sin \beta T_2 + \frac{\alpha}{\alpha^2+\beta^2} e^{-\alpha T_2} \cos \beta T_2 - \frac{\alpha}{\alpha^2+\beta^2} \right] \\ + I_o^1 \frac{1}{\beta} e^{-\alpha T_2} \sin \beta T_2 \end{cases}$$

Note from equations (1B) and (5B) that the amplitudes of i_1 and i_2 are directly dependent on Q_0 which is the negative of Q_1. Since all other quantities are constant, it can be [illegible]n that the amplitude of current is constant [illegible]m cycle to cycle if $Q_2 = Q_0$, and the amplitude builds up if $Q_2 > Q_0$. A relationship must be found among the circuit parameters and currents such that

$$Q_2 = Q_1 + \Delta Q = \Delta Q - Q_0 \geq Q_0$$

or

$$(8)\quad \Delta Q \geq 2Q_0$$

Substitute equation (7) into equation (8) and using $Q_0 = -Q_1$ in equation (7), extensive simplification yields:

$$\left(1 - \frac{I_o^1}{2I_s}\right) 2\alpha\beta \sin \beta T_2 - \beta^2 \frac{I_o^1}{2I_s}\left(e^{+\alpha T_2} - e^{-\alpha T_2} \right) \geq 0$$

$$\left(1 - \frac{I_o^1}{2I_s}\right) 2\alpha\beta \sin \beta T_2 - 2\beta^2 \frac{I_o^1}{2I_s} \sin h\ \alpha T_2 \geq 0$$

Since the second term will be positive,

$$\frac{\alpha}{\beta} \left(\frac{1 - \frac{I_o^1}{2I_s}}{\frac{I_o^1}{2I_s}} \right) \sin \beta T_2 \geq \sin h\ \alpha T_2$$

For convenience define $i_o^1 = \frac{I_o^1}{2I_s}$ so

$$(9)\quad \frac{\alpha}{\beta} \left(\frac{1 - i_o^1}{i_o^1} \right) \sin \beta T_2 \geq \sin h\ \alpha T_2$$

Looking at the relationship of equation or inequality (9) graphically illustrated in Figure 4, it is seen that the condition stated will exist for a range of the variable T_2 between 0 and T_m in Figure 4 provided the initial slope at $T_2 = 0$ of the left-hand term of equation (9) exceeds that of the right-hand term. This is necessarily satisfied since, referring to Figure 2a, I_o^1 must be less than I_s for resistance to occur at position i_v. The maximum value of i_o^1 is then 0.5, so the minimum value of

$$\frac{1 - i_o^1}{i_o^1}$$

is then unity. The initial slope of the right-hand term is alpha and that of the left-hand term must be greater than alpha, satisfying the condition for a positive range of T_2 in Figure 4 over which equation (9) holds.

It has now been found that the oscillation amplitude will build up if the duration of the resistive part of the cycle is less than a value T_m defined by

$$(10)\quad \frac{\alpha}{\beta} \left(\frac{1 - i_o^1}{i_o^1} \right) \sin \beta T_m = \sin h\ \alpha T_m$$

For given natural frequency, parameters, and supply currents, the value of T_2 increases as the amplitude of oscillation increases. For small oscillatory amplitudes such that $T_2 < T_m$, the amplitude grows; however, for larger oscillatory current amplitudes such that $T_2 > T_m$, the oscillatory current amplitude will decay. The [illegible]scillation current amplitude

then seeks a value such that $T_2 = T_m$ whereby a stable steady state amplitude of oscillation can exist without the necessity of switching the cryotron in the negative half cycle.

"Hunting" of the stable point, causing an oscillatory envelope, does not occur, if Q_2 is a monotonically increasing function of Q_o in the range of interest, as is the case here.

Now the stable amplitude of the positive and negative half cycles can be found as well as the period of oscillation as functions of the parameters.

The amplitude of the negative (free) half cycle, from (1B), is

$$\bar{I}_m = -\sqrt{(\omega_0 \; Q)^2 + (I_o^1)^2}$$

Where Q is the stable value of Q_o found from (6B) using (4) and $T_2 = Tm$.

The exact amplitude of the positive (driven) half cycle is found by obtaining the first maximum of equation (5B) by differentiation:

$$\overset{+}{I}_m = I_o^1 \sqrt{\frac{(a_0-\alpha)^2+\beta^2}{\alpha^2+\beta^2}} \left(e^{-\alpha T_p}\right)$$

$$\text{where } T_p = \sqrt{\frac{1}{\alpha^2+\beta^2}} - \frac{1}{\beta}\tan^{-1}\frac{\beta}{a_0-\alpha}$$

T_2 is already known, but the period of the free swing, T_1, must be found.

Referring to Figure 5, Equation (1B) indicates that the first positive $\omega_0 t$ intercept occurs at $\tan^{-1}$

$\left(\frac{I_o^1}{\omega_0 \; Q}\right)$, the second occurs at $\pi + \tan^{-1}\left(\frac{I_o^1}{\omega_0 \; Q}\right)$

and symmetry indicates that another addition of $\tan^{-1}$

$\left(\frac{I_o^1}{\omega_0 \; Q}\right)$ yields $\omega_0 T_1$.

Therefore, $\omega_0 \; T_1 = \pi + 2 \; \tan^{-1} \frac{I_o^1}{\omega_0 \; Q}$

$$(11) \quad T_1 = \frac{\pi}{\omega_0} + \frac{2}{\omega_0}\tan^{-1}\frac{I_o^1}{\omega_0 \, Q}$$

The period of oscillation is then $T = T_1 + T_2$

The specific case for which the cryotron is biased exactly at the switching point, that is, $I_o = I_b$ or $I_o^1 = i_c^1 = 0$, is of special interest since the relative simplicity of the solution allows for clearer physical interpretation. In addition, much of the preceding general analysis is irrelevant to this case. There follows a general outline of the specific results for $I_o^1 = 0$.

$$(1C) \quad i_1 = -\omega_0 \; Q_0 \sin \omega_0 t$$

$$(5C) \quad i_2 = \left(\frac{\alpha}{\beta} 2I_s + \frac{\omega_0^2}{\beta} Q_o\right) e^{-\alpha t} \sin \beta t$$

Note that (6A) is now trivial and (6B) is irrelevant.

For this case, T_2 is $\frac{\pi}{\beta}$, *not* a function of amplitude as it is in general.

$$Q_2 = Q_1 + \int_0^{\pi/\beta} i_2 \; dt = Q_1 + \Delta Q = -Q_o + \Delta Q$$

$$(7A) \quad \Delta Q = \left[\frac{\alpha}{\omega^2} 2I_s + Q_o\right] \left[1 + e^{-\left(\frac{\alpha}{\beta}\right)\pi}\right]$$

Again, the oscillation amplitude is building up if $Q_2 > Q_o$, stable if $Q_2 = Q_o$, and decaying if $Q_2 < Q_o$

$$(12) \quad Q_2 = 2I_s \frac{\alpha}{\omega_0^2}\left(1 + e^{-\frac{\alpha}{\beta}\pi}\right) + Q_o e^{-\frac{\alpha}{\beta}\pi}$$

Q_2 is a linearly increasing function of Q_o in this case, facilitating a graphical picture of the buildup of oscillation. There is a positive Q_2 intercept, so a slope less than one is required for a point to exist where $Q_2 = Q_o$. The slope $e^{-\frac{\alpha}{\beta}}$ satisfied this condition since $\frac{\alpha}{\beta}$ is always positive. Figure 6 shows Q_o and Q_2 each as a function of Q_o. The intersection is Q, the value of Q_o for which the oscillation amplitude is stable. Inspection reveals that oscillation will grow toward this point if initially below or decay toward it if initially above, and that no "hunting" can occur. The broken lines show the path of buildup in several successive cycles for an initial Q_o value of E.

Analytically, from (12)

$$(13) \quad Q = 2I_s \frac{\alpha}{\omega_0^2}\left[\frac{1 + e^{-\frac{\alpha}{\beta}\pi}}{1 - e^{-\frac{\alpha}{\beta}\pi}}\right]$$

The requirements of equations (26) are necessary but not sufficient to build up oscillation. In order to set further requirements, end conditions on the transmission line must be specified. Up until now reference has been made to an unloaded oscillator or one in which the far right end of the transmission line is truly an open circuit. In this case equation (26) is sufficient to describe buildup conditions. In practice, however, a load will be placed on the oscillator, probably at the far end of the line. Let us assume that this load is a pure resistance, R_o. The reflection coefficient K at the far end of the line is then

$$(27)\quad |K| = \frac{R_o - Z_o}{R_o + Z_o}$$

where we assume $R_o > Z_o$.

The current wave will be reflected twice from the load end before it will once again be required to produce a super-to-normal phase transition. The result when equation (26) is modified to account for loading is that

$$(28a)\quad |K|^2 \frac{I_s}{1+Z_o/R_g} \geq |I_\infty(F)| - I_b$$

and

$$(28b)\quad |K|^2 \frac{I_s}{1+Z_o/R_g} \geq \frac{G_1(I_\infty - I_b) - I_s}{G_1 - 1}$$

Combining equations (27) and (28) the starting conditions under load may be expressed as

$$(29a)\quad \frac{I_s}{1+Z_o/R_g} > \left(I_\infty(F) - I_b\right)\left(\frac{R_o+Z_o}{R_o-Z_o}\right)^2$$

$$(29b)\quad \frac{I_s}{1+Z_o/R_g} > \left[\frac{G_1(I_\infty - I_b) - I_s}{G_1 - 1}\right]\left[\frac{R_o+Z_o}{R_o-Z_o}\right]^2$$

Equation (29) seems to indicate that it is theoretically possible to compensate for almost any load resistor greater than (but not equal to) Z_o by adjusting the bias. Practically, a limit exists on just how close the bias may be set near the gain curve; a difference of a few micro amperes between I_∞ (F) and I_b may not be feasible.

Equation (29) can also be used to determine the minimum value that R_o can have and yet allow oscillation to begin with a given set of parameters.

The minimum load that can be used under given conditions is

$$(30)\quad R_o > Z_o \frac{\sqrt{\dfrac{\frac{I_s}{I_\infty(F) - I_b}}{1+Z_o/R_g} + 1}}{\sqrt{\dfrac{\frac{I_s}{I_\infty(F) - I_b}}{1+Z_o/R_g} - 1}}$$

The assumption of instantaneous switching of the gate implies that the steps in the buildup of current will be of uniform amplitude. In practice this does not happen and the steps of additional current will tend to decrease during the buildup.

The amplitude will continue to increase until one of the limiting actions takes place. The maximum amplitude, S, possible under conditions of limiting at point E is

$$(31a)\quad S(E) = |I_b| - |I_\infty(E)|$$

or

$$(31b)\quad S(E) = \frac{G_2(I_b + I_\infty) - I_s}{G_2 + 1}$$

The amplitude obtainable for limiting at point Q is

$$(32)\quad S(Q) = |I_\infty(Q)| - I_b = I_s$$

A large swing may be obtained by selecting an operating point such that

$$(33)\quad S(E) = S(Q) = I_s$$

This point is defined by

$$(34a)\quad I_s = I_b - |I_\infty(E)|$$

or

$$(34b)\quad I_s = \frac{G_2(I_b + I_\infty)}{G_2 + 2}$$

The number of cycles required to build up to steady-state oscillation can be found approximately by dividing the final amplitude by the current addition per step.

Thus when operating according to the conditions specified by equation (33), the number of cycles, n, is given approximately by

$$(35)\quad n \approx \frac{1}{2}\left[1 + \frac{Z_o}{R_g}\right]$$

The power that the oscillator delivers to the load is given by

$$(36)\quad P_{out} = \frac{1}{2}(1 - |K|)^2 S^2 R_e$$

which under the conditions specified by equation (33) is

$$(37)\quad P_{out} = 2\, I_s^2 Z_o \frac{Z_o/R_e}{(1+Z_o/R_g)^2}$$

A possible source of trouble is having a nonuniform wave front of current propagating on the transmission line. This arises since there is a phase difference between the current wave reflected from the superconducting gate and the additional step of current produced when the gate switches resistive. To minimize this effect it is necessary to keep the distance between the gate-ground connection and the control line down to a minimum. This length of line is longer in the case of an in-line cryotron than in the crossed-film cryotron. The effect is accentuated further by the finite time required for resistance to appear. Both of these difficulties may contribute to the ultimate frequency limitation of this device.

Another problem that may arise is the presence of mismatches caused by discontinuities in the transmission line. Calculations show that within the expected range of discontinuities, the additional reflections have negligible effect.

An oscillator was designed with the following dimensions.

Cryotron	— In-line
Gate	— Indium — 4000 Å, 0.009 × $\frac{3}{16}$"
R_g	= 0.054 ohms
Bias field	= 116 oersteds
I_s	= 0.280 amp
T/T_c	= 0.840
Gain (G_1)	= 4
$I_\infty(F) - I_b$	= 0.0125 amp
Transmission Line	
Z_o	= 0.432 ohms
velocity	= C/2
length	= $2\frac{1}{2}$"
$\frac{I_s R_g}{R_g + Z_o}$	= 0.031 amp
Oscillator	
Frequency	= 0.57 Gc/s
Amplitude	= 0.255 amp
Minimum R_e	= 2 ohms
Power Out	R_e = 50 ohms; 0.49 milliwatt R_e = 300 ohms; 0.08 milliwatt
Dissipation in the Gate Element	$= \frac{1}{2} I_s^2 R_g = 2.1$ milliwatts

A glass substrate is not useable with this high dissipation level since the temperature rise increases the operating temperature to greater than .90 per cent of the critical temperature. At this high operating temperature there is little or no gain, and furthermore, the gate element is always resistive at the operating current levels. To circumvent this, a high thermal conductivity substrate is required, such as sapphire or aluminum.

WHAT WE CLAIM IS:—

1. An oscillator circuit including at least one cryotron having a gate conductor and a control conductor, in which the gate conductor is connected across a source of driving current in parallel with a series circuit including the control conductor and a capacitance, and including means for applying a bias magnetic field to the gate conductor.

2. A circuit as claimed in claim 1, in which

the capacitance is provided by the distributed capacitance of a transmission line.

3. An oscillator as claimed in claim 2, in which the transmission line is open-ended and is one quarter wavelength long at the desired frequency of oscillation of the circuit.

4. An oscillator circuit including a cryotron having a gate conductor and a control conductor; a source of current connected across two paths in parallel with each other, in one of which is the gate conductor of the cryotron and in the other of which is the control conductor of the cryotron in series with a capacitor and inductance, and in which the control winding has connected in parallel across it a source of bias current.

5. An oscillator circuit including two cryotrons each having a gate conductor and a control conductor, in which the gate conductors are connected in parallel with each other across a source of driving current, in which one end of one control conductor is connected through a series-connected capacitor and inductor with one end of the other control conductor, in which the other end of each of the control conductors is connected directly or indirectly to the source of driving current, and including means for applying a bias magnetic field to the gate conductor of each cryotron.

6. A circuit as claimed in claim 5, in which the biassing means includes a source of bias current connected in parallel with the respective control conductor.

7. An oscillator circuit substantially as described herein with reference to Figures 1, 7, 8, 9 or 10 of the accompanying drawings.

For the Applicants,
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Chartered Patent Agent.

Leamington Spa: Printed for Her Majesty's Stationery Office, by the Courier Press (Leamington) Ltd.—1965. Published by The Patent Office, 25 Southampton Buildings, London, W.C.2, from which copies may be obtained.

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5 SHEETS *This drawing is a reproduction of the Original on a reduced scale*
Sheet 1

FIG. 1.

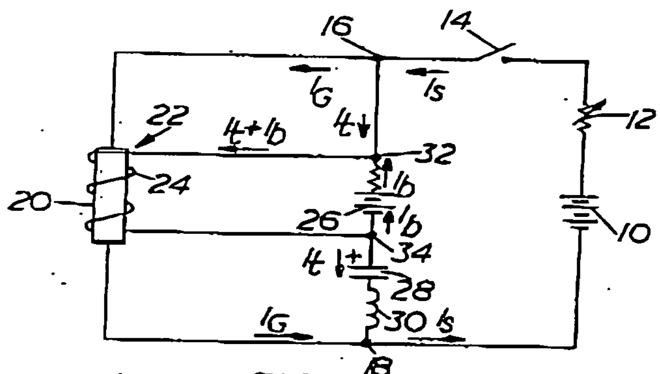
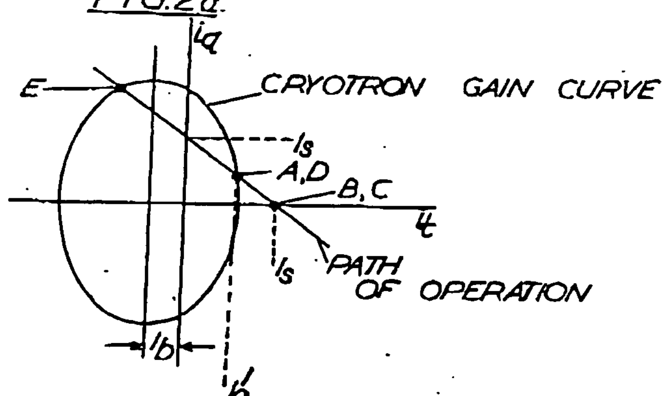


FIG. 2a.



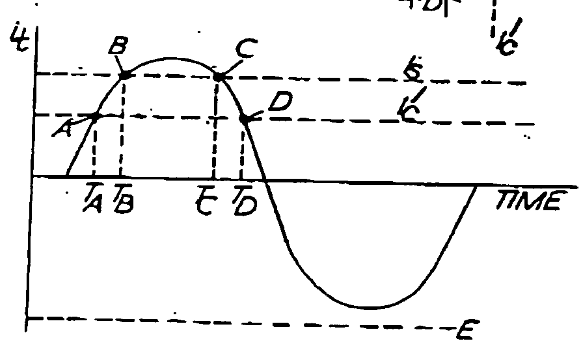


FIG. 2b.

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FIG. 3.

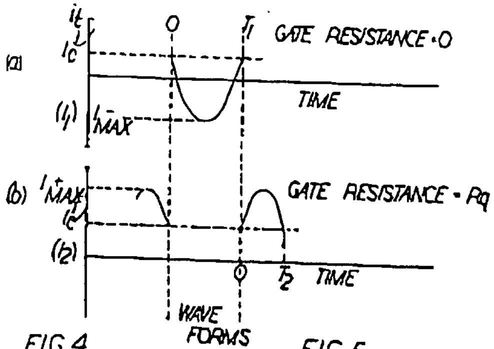

FIG. 4.

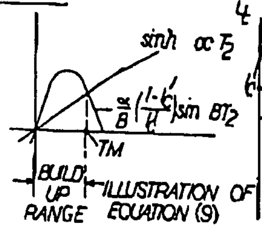

FIG. 5.

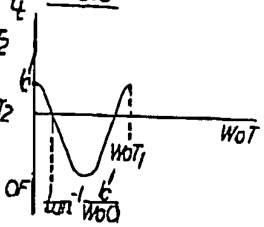

FIG. 6.

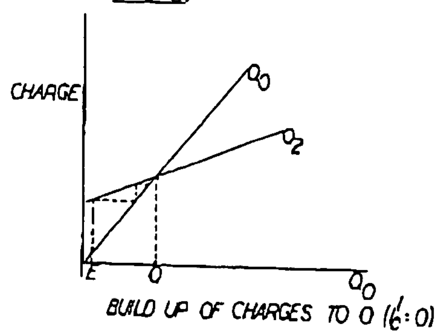

FIG 7

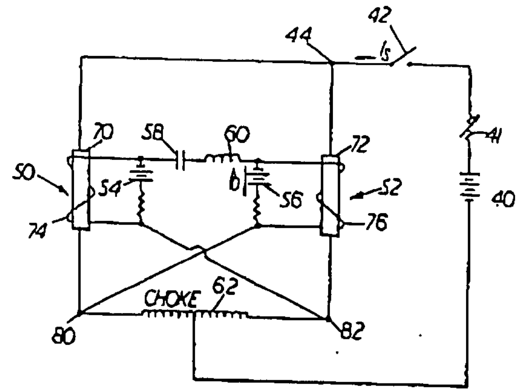

FIG. 8.

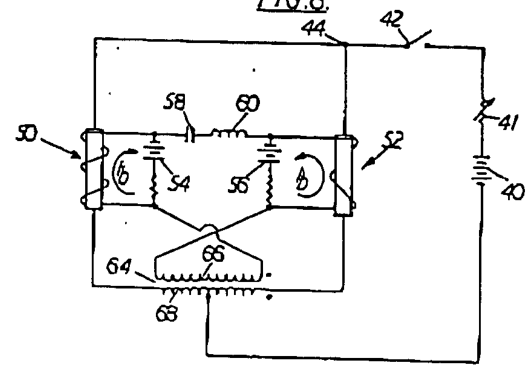

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FIG.9.

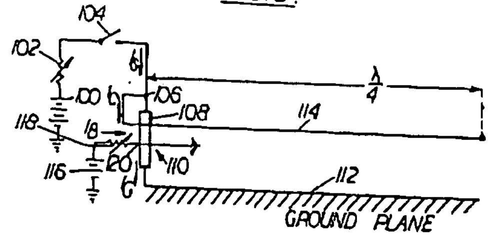

FIG.10.

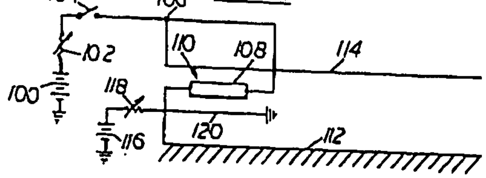

FIG.11

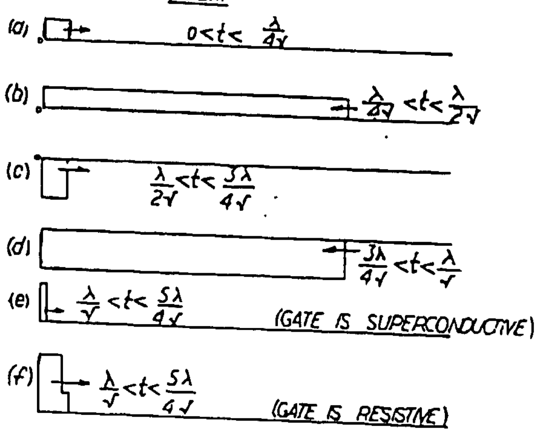

FIG.12.

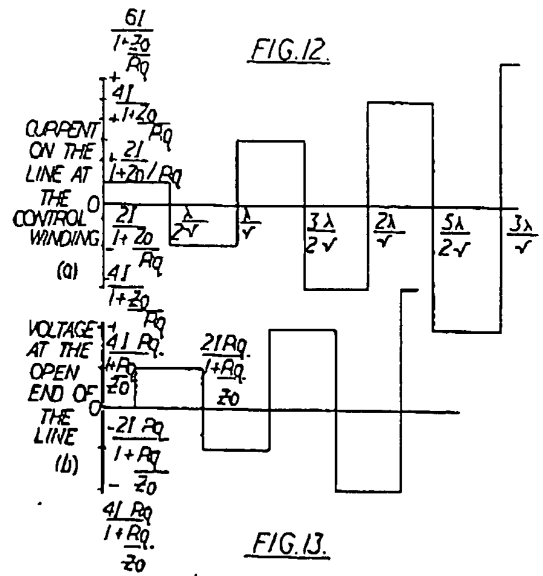

FIG.13.